A GENERAL PROGRAM FOR STEADY STATE, STABILITY, AND FM NOISE ANALYSIS OF MICROWAVE OSCILLATORS

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ABSTRACT

This paper presents two new algorithms for nonlinear autonomous circuits CAD.

In the first, a symbolic simulator is used to calculate the possible oscillations frequencies of the circuit, then the high level behaviour of the oscillator is determined by the harmonic balance method extended to autonomous circuits.

The second algorithm is based on the conversion matrix method, which allows simulation of nonlinear microwave oscillators phase and amplitude noise spectra with linear and nonlinear correlated noise sources.

I - INTRODUCTION

The design of an oscillator requires the use of a set of analysis which must characterize completely the electrical behaviour of the circuit.

Those calculations are :

* determination of all frequencies where oscillations may start. After modification of the topology, verification that oscillations may only start at a frequency close to the desired one.
* détermination of the accurate steady state and the oscillation frequency f_0, (waveforms, power and harmonics).
* study of the stability of the solution under bias and the variation value of an element.
* Evaluation of FM noise spectrum around ω_0 and eventually around $n.\omega_0$ (case of multiplier-oscillator).

Actually, those different steps have been separately and incompletely studied. Methodology and tools we present are based on the spectral balance method. The unicity of formalism used in each step ensures coherence of the set, and makes its integration in general nonlinear CAD software easier. The now classical spectral balance method requires solving the nonlinear system of equations (1) in the frequency-domain [1,2,3,4]

$$F(C) = C - A_1\, NL - A_2\, E_g = 0 \qquad (1)$$

where C and NL are split vectors of real and imaginary parts of control variables and nonlinear variables, respectively, Eg is the external source vector, and A_1 and A_2 are matrices representing linear parts of the circuit, at $0, \omega_0 ...., N.\omega_0$. The iterative process leading to a solution (Newton Raphson method) involves a Jacobian évaluation.

$$J = 1 - A_1 . U \quad (2) \qquad \text{with } U = \frac{\partial NL_j^k}{\partial C_i^l} \quad (3)$$

$i = 1$, number of control variables

$j = 1$, number of nonlinear variables

$l = 0, N$

$k = 0, N$

It will be shown in next sections that all oscillator analysis steps require the use of that derivative matrix U. This methodology has been applied to the design of a dielectric resonator oscillator. This circuit is shown figure 1.

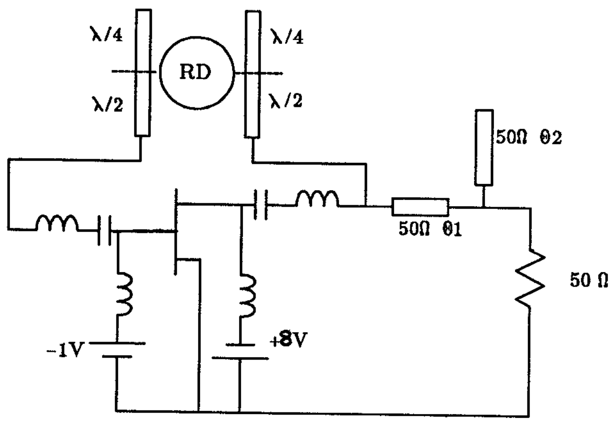

Figure 1 : Analysed oscillator circuit

II - CALCULATION OF THE STEADY STATE BEHAVIOUR

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A - Starting frequency range

For a given oscillator, finding the steady state (frequency, electrical state) by an iterative process requires an initial frequency estimation as close as possible to the accurate solution. Robustness and convergence speed of the process depend on this estimation.

To achieve this goal, the circuit stability is studied, around DC [4] by drawing the graph of the locus of the determinant for the following linear system :

$I - A_1(\omega)\,U$ (4), with $U = \dfrac{\partial NL_i^0}{\partial C_j^0}$ (5)

and $\omega : 0 \rightarrow \infty$

If the Nyquist locus involves the origin, that is to say if the circuit may oscillate, two frequencies: f_{inf} and f_{sup}, near the oscillation frequency may be defined. For the circuit analysed here, the graph of :

$\det(I - A_1(\omega)\,U)$ *is shown in figure 2 and the starting frequency range is bounded by :*

$f_{inf} = \begin{cases} Re(det) = 0 \\ Im(det) < 0 \end{cases}$ and by $f_{sup} = \begin{cases} Re(det) < 0 \\ Im(det) = 0 \end{cases}$

This method necessitates the evaluation of matrix A_1, for a set of f values between DC and 100 GHz. Also, for speed improvement, this matrix has been generated by means of symbolic analysis, from a bloc circuit description. The symbolic calculus will also be used later to calculate the derivatives versus frequency which are necessary in the iterative process.

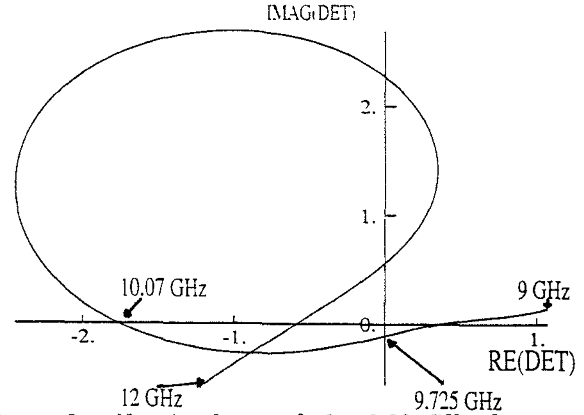

***Figure 2** : Nyquist locus of the 9,98 GHz frequency oscillator, Locus 9 GHz $\leqslant f \leqslant$ 12 GHz ; V_{gso} = - 1 V, V_{dso} = 8 V*

B - Steady state search

The spectral balance equation for a non autonomous circuit is written as :

$$F(C) = C - A_1 NL - A_2 Eg = 0 \qquad (6)$$

$$\text{with } C = C_1^{R,0}, C_1^{I,0}, C_1^{R,1}, C_1^{I,1} \ldots, C_i^{R,k}, C_i^{I,k}, \ldots, C_M^{R,N}, C_M^{I,N} \qquad (7)$$

where R,I denote real or imaginary part.

i = 1, M is the control variable number

k = 0, N is the harmonic number

This equation must be modified as follows to allow autonomous circuit analysis:

- The Eg vector contains only DC generators so it is necessary to introduce frequency as an unknown.

- In order to keep a particular steady state a phase origin must be chosen.

The new system of equations becomes:

$$F(\bar{C}, \omega) = 0$$

with :

$$\bar{C} = C_1^{R,0}, C_1^{I,0}, C_1^{R,1}, C_1^{R,2} \ldots C_M^{R,N}, C_M^{I,N}$$

fixing $C_1^{I,1} = 0$

*It must be noticed that the new Jacobian contains terms in the form of $\dfrac{\partial A_1}{\partial \omega}$**NL.** The evaluation of those terms implies matrix A_1, sensitivity evaluation with respect to ω. So, at each iteration, it is necessary to evaluate $A_1(\omega)$ and $\dfrac{\partial A_1(\omega)}{\partial \omega}$. By means of symbolic analysis this step can be achieved without time penalty. For the circuit of interest, the resulting steady state waveforms and its harmonics are given in figure 3.*

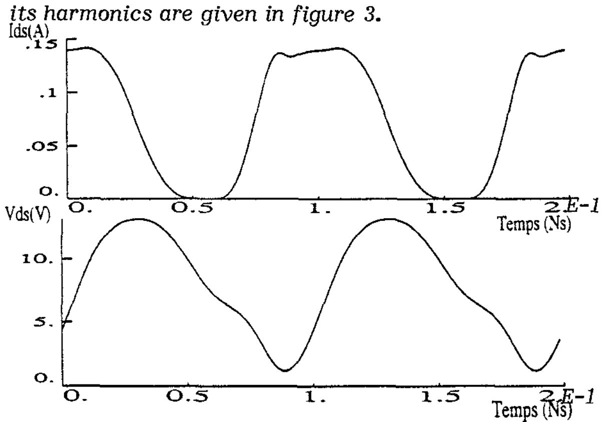

***Figure 3** : Time-domain waveforms obtained by simulation (f_0 = 9,98 GHz)*

III - NOISE ANALYSIS IN NON-LINEAR CIRCUITS

The noise theory in free running and synchronised oscillators with negative resistance has been established by Kurokawa who combined previous works in a convenient form to calculate both microwave oscillator phase and amplitude noise spectra. Results are limited to the calculation of noise due to linear noise sources.

Different improvements have been proposed using noise generator representation established by Penfield [5] and using the conversion matrix formalism. Kerr and Held [6] have extended the noise analysis to diode mixers with nonlinear noise sources. Recently the method has been applied to FET mixers [7,8]. Conversion matrix formalism has also been used to calculate the AM and FM noise in FET oscillators. However, the calculation assumes a simple circuit and linear noise sources. In this presentation, we propose a general tool to analyse the noise conversion in nonlinear circuits. It allows calculation FM and AM noise of nonlinear oscillators as well as noise figure of mixers.

This algorithm has been run while linked to the harmonic balance algorithm previously discussed in the first part of this paper. The number of semiconductor devices used in the circuit under analysis is not limited. All noise sources may be nonlinear and correlated. The algorithm calculates the noise voltage and power resulting from nonlinear conversion in any branch of the circuit. Then the noise spectrum may be easily deduced. To perform the noise analysis, the circuit is represented by figure [4].

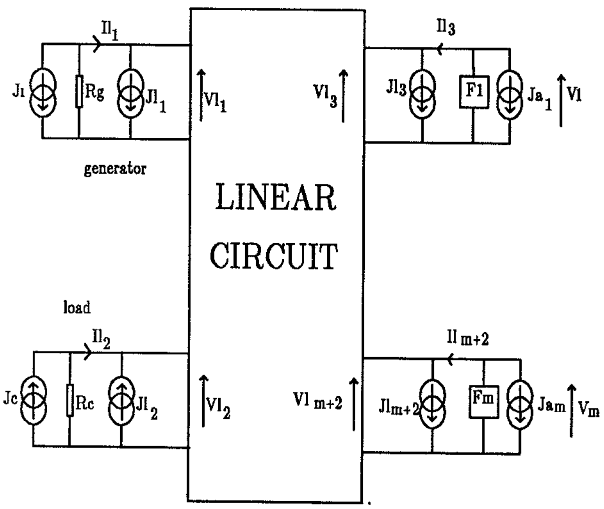

Figure 4 : General equivalent representation of a noisy circuit. The J_l are linear noise current sources at the linear subnetwork ports. The J_α are nonlinear noise current sources connected across the nonlinearities. J_c, J_i are thermal noise sources of generator and load.

In this figure, two subnetworks appear. The first is the nonlinear subcircuit which is a gathering of nonlinearities modelling active devices such FET, bipolar transistor, diodes, ... characterized by their conversion matrices calculated with the help of the U matrix of eq(3). The second is the linear subnetwork which includes active device parasitic elements and the remaining circuit. Two accesses must be pointed out :

- The external generator access-port (for mixer and synchronised oscillator analysis).
- The load access where the resulting noise voltages will be calculated.

The noise sources are separated in linear and nonlinear generators and are represented by their correlation matrices.

- The linear noise sources depend on the topology and temperature of the circuit.
- The nonlinear sources depend also on the semiconductor bias and on the oscillation-signal. This signal has been previously calculated by the harmonic-balance algorithm described in the first part of the paper.

The purpose of the noise analysis is to find first the noise voltages converted around each harmonic-frequency of the fundamental oscillation frequency f_o, at a distance from the carrier. The noise voltage in a branch may be represented by a vector V :

$$\tilde{V} = \begin{bmatrix} V_{kl}(k\omega_o - \Omega) \\ \vdots \\ V_{1l}(\omega_o - \Omega) \\ V(\Omega) \\ V_{1u}(\omega_o + \Omega) \\ \vdots \\ V_{ku}(k\omega_o + \Omega) \end{bmatrix}$$

- The phase noise spectrum at a distance Ω from $k\,\omega_o$ is defined as :

$$S_{k\phi} = \frac{\overline{|\tilde{V}_{ku}|^2} + \overline{|\tilde{V}_{kl}|^2} - 2\,Re\,\overline{|\tilde{V}^*_{ku}\tilde{V}^*_{kl}\,e^{j2\varphi_o}|}}{V_o^2}$$

- The amplitude noise spectrum is defined as :

$$S_{kA} = \frac{\overline{|\tilde{V}_{ku}|^2} + \overline{|\tilde{V}_{kl}|^2} + 2\,Re\,\overline{|\tilde{V}^*_{ku}\tilde{V}^*_{kl}\,e^{j2\varphi_o}|}}{V_o^2}$$

- The amplitude-phase noise correlation spectrum is :

$$S_{kA\phi} = -S^*_{kA\phi} = j\,\frac{\overline{|\tilde{V}_{ku}|^2} - \overline{|\tilde{V}_{kl}|^2} + 2j\,Im\,\overline{|\tilde{V}^*_{ku}\tilde{V}^*_{kl}\,e^{2j\varphi_o}|}}{V_o^2}$$

where : $V_o e^{j\varphi_o}$ is the carrier voltage at $k\,\omega_o$, in the branch where the noise is calculated. These expressions are calculated at the output load access of the oscillator.

First the equivalent two port [Y_{eq}] : between the external generator and the output load is determined, and the noise sources are extracted at these ports.

Then the correlation matrice between noise voltages of the external generator port V_i (for synchronized oscillators) and the load port : V_c , is calculated as:

$$C = \begin{vmatrix} V_i \\ \\ V_c \end{vmatrix} \cdot | V_i \ V_c |^* $$
$$= Z_L \{ H_A \, C_{JA} \, H_A^{*t} + H_L \, C_{jL} \, H_L^{*t} + C_j \} \, Z_L^{*t}$$

where Z_L, H_A, H_L are matrices dependent on the topology of the circuit and on the conversion matrices of the nonlinear elements.

C_{jA} is the correlation matrix of the nonlinear noise sources of the circuit

C_{jL} is the correlation matrix of the linear noise sources of the circuit

C_j is the correlation matrix of the noise sources of the external generator and output load.

From that equation, the noise spectrum at the output port may be calculated.

IV - EXAMPLE OF NOISE ANALYSIS

As an example of noise analysis, three noise voltage generators are introduced in the FET gate. The first at low frequencies, varies as 1/f ; the two others as symetrical sideband of the oscillation frequency , have constant thermal noise spectral density. The phase noise spectrum obtained by the simulation is shown in figure 5. Near the carrier, the slope is -30 dB/dec which confirms 1/f noise conversion. The converted thermal noise is characterized by a 20 dB/dec slope far from the carrier. These results are classical in oscillator phase noise spectra.

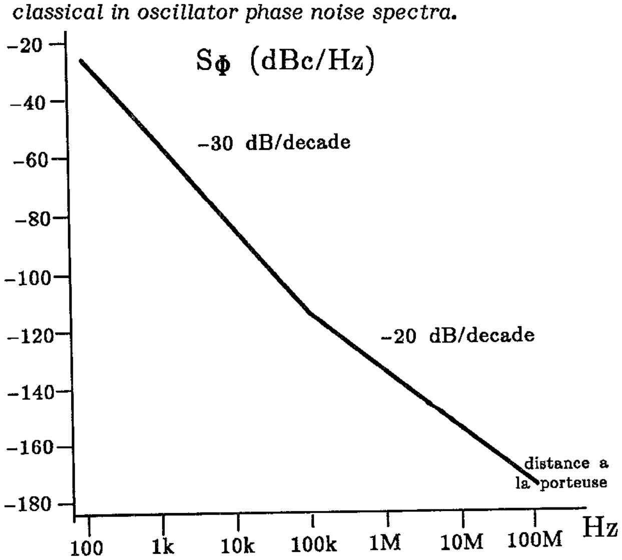

***Figure 5** : Simulated phase noise spectrum with :*

$e^2_{1/f} = \frac{10^{-10}}{f} \ \mu v^2/H_z$; $e^2_{th} = 10^{-4} \ \mu v^2/H_z$

CONCLUSION

In this article, we have introduced two new algorithms enabling analysis of the oscillators (in high signal for the prediction of level) the conditions of oscillation (frequency, amplitude) and their behaviour in noise (phase and amplitude noise spectra density). The first set of programs is a direct extension of the harmonic balance method to the analysis of autonomous circuits in which the frequency is unknown. This analysis enables one to search for the oscillation frequency and the steady state of every nonlinear circuit from the initial pre-estimated conditions. It is to be noted that the design of an oscillator for a given frequency is also possible with this method. In this case, a frequency variable is replaced by a variable "an element of linear circuit". The conditions of harmonic balance are then checked for the desired frequency. The second algorithm uses the same formalism for the analysis of the parametrical circuits which is based on the characterization of nonlinearities by conversion matrices. This enable one to find the phase and amplitude noise spectrum of this oscillator using a simple linear analysis. This method can also be used in the analysis of the conversion gain and the mixer noise figure.

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